Assistants de preuve et systèmes dynamiques

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1. The challenge

I want to propose a challenge: Formalize the proof of the following theorem.

Theorem 1.1 *(Clausen-S.) Let $0 < p' < p \leq 1$ be real numbers, let S be a profinite set, and let V be a p-Banach space. Let $\mathcal{M}_{p'}(S)$ be the space of p'-measures on S. Then*

$$\mathrm{Ext}^i_{\mathrm{Cond(Ab)}}(\mathcal{M}_{p'}(S), V) = 0$$

for $i \geq 1$.

(This is a special case of Theorem 9.1 in www.math.uni-bonn.de/people/scholze/Analytic.pdf, and is the essence of the proof of Theorem 6.5 there.)

Below, I will explain what all the terms in the theorem are, and why I care. I apologize in advance that the background story is a little longer, but I think it's a fascinating story and I tried to be as brief as possible.

A quantitative version of the Morse lemma and quasi-isometries fixing the ideal boundary

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Abstract

The Morse lemma is fundamental in hyperbolic group theory. Using exponential contraction, we establish an upper bound for the Morse lemma that is optimal up to multiplicative constants, which we demonstrate by presenting a concrete example. We also prove an "anti" version of the Morse lemma. We introduce the notion of a geodesically rich space and consider applications of these results to the displacement of points under quasi-isometries that fix the ideal boundary.

A CORRECTED QUANTITATIVE VERSION OF THE MORSE LEMMA

SÉBASTIEN GOUËZEL AND VLADIMIR SHCHUR

Abstract. There is a gap in the proof of the main theorem in the article [Shc13a] on optimal bounds for the Morse lemma in Gromov-hyperbolic spaces. We correct this gap, showing that the main theorem of [Shc13a] is true. We also describe a computer certification of this result.

2. Subadditive ergodic cocycles

In this section, we prove Theorem 1.1. By passing to the natural extension if necessary, one can assume without loss of generality that T is invertible.

2. Subadditive ergodic cocycles

In this section, we prove Theorem 1.1. First note that we can assume that the space (Ω, μ) is a standard Lebesgue space. Indeed, since the statement of Theorem 1.1 only deals with the distribution of the countable family of real-valued functions $(a(n, T^k\omega))_{n,k\in\mathbb{N}^2}$, it suffices to work on the space $\mathbb{R}^{\mathbb{N}^2}$ with the Borel probability measure encoding the joint distribution of all these functions and the shift map. This space is a standard Lebesgue space. Then, passing to the natural extension if necessary, one can assume without loss of generality that T is invertible. (Natural extensions exist in general only for transformations of a standard Lebesgue space, which is why the first reduction was needed).

```
theorem birkhoff_theorem_AE_nonergodic:
  fixes f::"'a ⇒ real"
  assumes "integrable M f"
  shows "AE x in M. (λn. birkhoff_sum f n x / n) ⟶ real_cond_exp M Invariants f x"
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LEAN Community

Lean and its Mathematical Library

The Lean theorem prover is a proof assistant developed principally by Leonardo de Moura.

The community recently switched from using Lean 3 to using Lean 4. This website is still being updated, and some pages have outdated information about Lean 3 (these pages are marked with a prominent banner). The old Lean 3 community website has been archived.

The Lean mathematical library, *mathlib*, is a community-driven effort to build a unified library of mathematics formalized in the Lean proof assistant. The library also contains definitions useful for programming. This project is very active, with many regular contributors and daily activity.

AI achieves silver-medal standard solving International Mathematical Olympiad problems

25 JULY 2024

AlphaProof and AlphaGeometry teams

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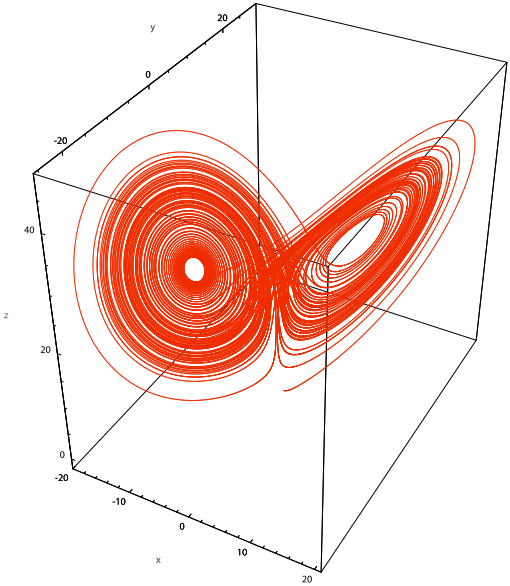
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A Rigorous ODE Solver and Smale's 14th Problem

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Abstract. We present an algorithm for computing rigorous solutions to a large class of ordinary differential equations. The main algorithm is based on a partitioning process and the use of interval arithmetic with directed rounding. As an application, we prove that the Lorenz equations support a strange attractor, as conjectured by Edward Lorenz in 1963. This conjecture was recently listed by Steven Smale as one of several challenging problems for the twenty-first century. We also prove that the attractor is robust, i.e., it persists under small perturbations of the coefficients in the underlying differential equations. Furthermore, the flow of the equations admits a unique SRB measure, whose support coincides with the attractor. The proof is based on a combination of normal form theory and rigorous computations.

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A Verified ODE Solver and the Lorenz Attractor

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Abstract A rigorous numerical algorithm, formally verified with Isabelle/HOL, is used to certify the computations that Tucker used to prove chaos for the Lorenz attractor. The verification is based on a formalization of a diverse variety of mathematics and algorithms. Formalized mathematics include ordinary differential equations and Poincaré maps. Algorithms include low level approximation schemes based on Runge–Kutta methods and affine arithmetic. On a high level, reachability analysis is guided by static hybridization and adaptive step-size control and splitting. The algorithms are systematically refined towards an implementation that can be executed on Tucker's original input data.